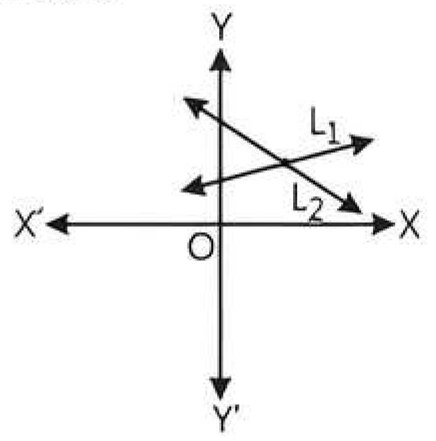
3 Pair of Linear Equations in Two Variables

Fastrack Revision

- **Linear Equation in Two Variables:** An equation of the form $ax + by + c = 0$, where a, b and c are real numbers ($a \neq 0$, $b \neq 0$), is called a linear equation in two variables x and y, *e.g.*, $7x - 3y - 2 = 0$, $4x + 9y = 14$, etc.
- **Solution of Linear Equation in Two Variables:** The values of the variables which satisfy the given linear equation is called solution of the linear equation. So, (x, y) is the solution of the linear equation $ax + by + c = 0$.
- **Simultaneous Linear Equations in Two Variables:** Two linear equations in two unknown variables x and y are said to form a system of simultaneous linear equations, if each of them is satisfied by the same pair of values of x and y.
The general form of a pair of linear equations is

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

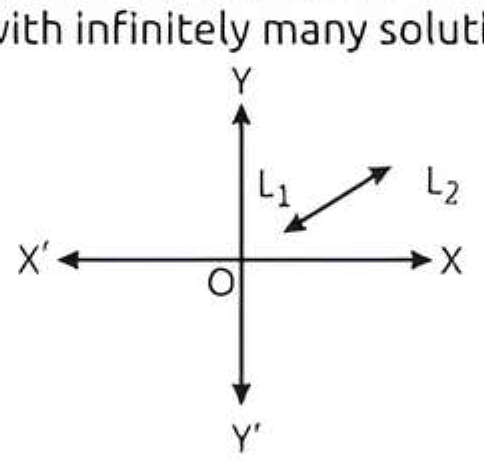
where a_1, a_2, b_1, b_2, c_1, c_2 are real numbers, such that $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$.
(*i.e.*, it means either a_1 or b_1 may be zero, but not both of them together should be zero. Similar condition exist for a_2 and b_2).
- **Consistent and Inconsistent Systems of Linear Equations:**
  1. **Consistent System of Linear Equations:** A system of two linear equations in two unknown variables is said to be consistent, if it has at least one solution.
  Suppose two linear equations are $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.
  Case I: When two lines intersect each other at one point, then system of equations is called consistent with unique solution.



One (Unique) solution

Condition for Unique Solution: $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

Case II: When two lines are coincident, *i.e.*, overlap each other, then system of equations is called consistent (dependent) with infinitely many solutions.

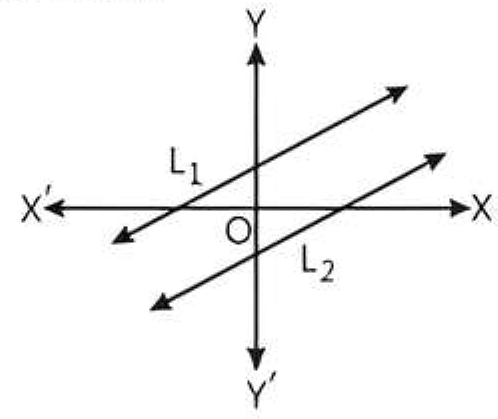


Infinitely many solutions

Condition for Infinitely Many Solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

  2. **Inconsistent System of Linear Equations:** A system of two linear equations in two unknown variables is said to be inconsistent, if it has no solution at all, *i.e.*, lines are parallel to each other.



No solution

Condition for No Solution:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

- **Methods of Solving Linear Equations in Two Variables:** There are two methods to represent and solve these equations:
  1. **Graphical Method:** A pair of linear equations is represented by two lines in a graph, which are shown in above graphs.
  2. **Algebraic Method:** It includes two methods to find the solution of a pair of linear equations:
     (i) Substitution Method: If we have a pair of linear equations in two variables x and y, then we have to follow certain steps to solve them using substitution method:
     Step 1: Find the value of one variable, say y in terms of the other variable, *i.e.*, x from either equation whichever is convenient.
     Step 2: Substitute this value of y in the other equation and reduce it to an equation in one variable, *i.e.*, in terms of x, which can be solved.
     Step 3: Substitute the value of x (or y) obtained in step 2 in the first equation and obtain the value of other variable.
     Note: Sometimes while solving step 2, you can get statements with no variable. If this statement is true, you can conclude that the given pair of linear equations has infinitely many solutions. If the statement is false, then the pair of linear equations is inconsistent.
     (ii) Elimination Method: Study the steps to be followed in the elimination method:

Step 1: Multiply both the equations by some suitable non-zero constant to make the coefficient of one variable numerically equal.

Step 2: Add or subtract one equation from the other so that one variable gets eliminated.

Step 3: Solve the equation in one variable so obtained to get its value.

Step 4: Substitute the calculated value of variable in the given equations to find the value of the other variable.

Note:

- If in step 2, we obtain a true statement involving no variable, then the pair of linear equations has infinitely many solutions.
- If in step 2, we obtain a false statement involving no variable, then the pair of linear equations has no solution, *i.e.*, it is inconsistent.

Knowledge BOOSTER

1. *Sometimes, a pair of equations in two variables are not linear but can be reduced to linear form by making some suitable substitutions. Here, first we find the solution of new pair of linear equations and then find the solution for the given pair of equations.*
2. *Suppose speed of boat in still water be x km/h and speed of stream be y km/h. Then the speed of boat in downstream is $(x + y)$ km/h and speed of boat in upstream is $(x - y)$ km/h.*

Practice Exercise

Multiple Choice Questions

Q 1. Which of the following method(s) is/are used to find the solution of a pair of linear equations algebraically?

a. Substitution method

b. Elimination method

c. Either substitution or elimination method

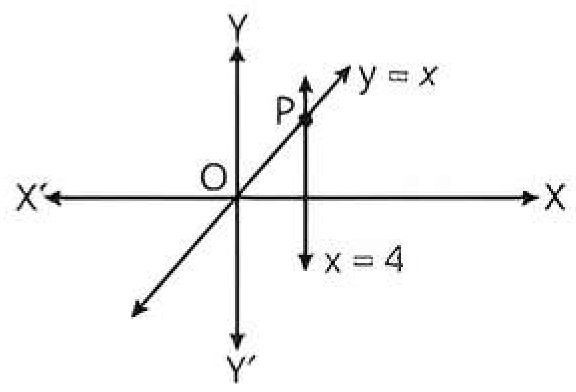
d. Both substitution and elimination methods

Q 2. The larger of two supplementary angles exceeds the smaller by 18 degrees. What is the measure of larger angle? [CBSE 2023]

a. 81° b. 99°

c. 36° d. 54°

Q 3. The lines represented by the linear equations $y = x$ and $x = 4$ intersect at P. The coordinates of the point P are: [CBSE 2023]



a. (4, 0) b. (4, 4)

c. (0, 4) d. (−4, 4)

Q 4. The point of intersection of the line represented by $3x - y = 3$ and Y-axis is given by: [CBSE 2023]

a. (0, −3) b. (0, 3)

c. (2, 0) d. (−2, 0)

Q 5. The values of x and y satisfying the two equations $32x + 33y = 34$, $33x + 32y = 31$ respectively are: [CBSE 2021 Term-I]

a. −1, 2 b. −1, 4

c. 1, −2 d. −1, −4

Q 6. The pair of equations $y = 0$ and $y = -7$ has: [CBSE SQP 2023-24]

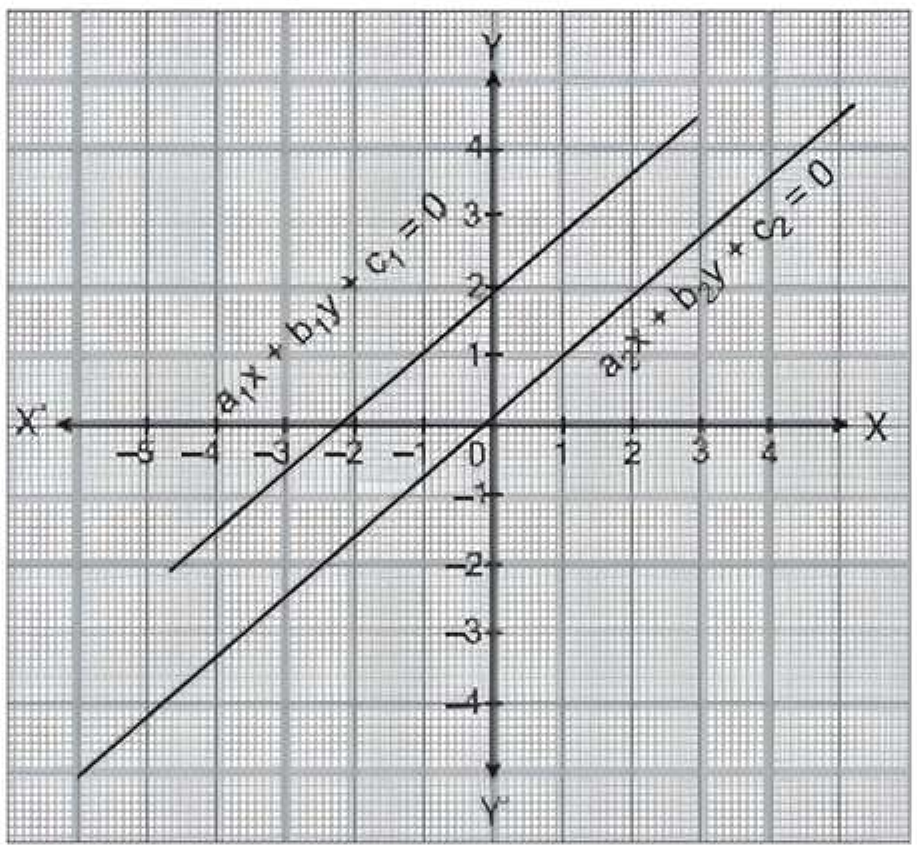
a. one solution

b. two solutions

c. infinitely many solutions

d. no solution

Q 7. The given pair of linear equations is non-intersecting. Which of the following statements is true? [CBSE SQP 2023-24]



a. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ b. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

c. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$ d. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Q 8. The pair of equations $x = a$ and $y = b$ graphically represents lines which are: [CBSE 2023, CBSE SQP 2021 Term-I]

a. parallel b. intersecting at (b, a)

c. coincident d. intersecting at (a, b)

Q 9. The pair of equations $2x - 3y + 4 = 0$ and $2x + y - 6 = 0$ has:

a. a unique solution b. exactly two solutions

c. infinitely many solutions d. no solution

Q 10. The pair of linear equations $x + 2y - 5 = 0$ and $2x - 4y + 6 = 0$: [CBSE 2023]

a. is inconsistent
b. is consistent with many solutions
c. is consistent with a unique solution
d. is consistent with two solutions

Q 11. The pair of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are: [CBSE 2023]

a. intersecting
b. parallel
c. coincident
d. either intersecting or parallel

Q 12. If the pair of equations $3x - y + 8 = 0$ and $6x - ry + 16 = 0$ represent coincident lines, then the value of r is: [CBSE 2023]

a. $-\frac{1}{2}$ b. $\frac{1}{2}$
c. -2 d. 2

Q 13. The value of k for which the lines $5x + 7y = 3$ and $15x + 21y = k$ coincide is: [CBSE SQP 2021 Term-I]

a. 9 b. 5 c. 7 d. 18

Q 14. If the system of equations $2x + 3y = 6$ and $2ax + (a + b)y = 24$ has infinitely many solutions, then:

a. $a = 2b$ b. $b = 2a$
c. $a + 2b = 0$ d. $2a + b = 0$

Q 15. If the pair of equations $x + y = \sqrt{2}$ and $x \sin\theta + y \cos\theta = 1$ has infinitely many solutions, then θ is equal to:

a. $30°$ b. $45°$ c. $60°$ d. $90°$

Q 16. If $5x - 3y = 9$ and $(a - b)x - (a + b - 3)y = a - 4b$ represent coincident lines, then the values of a and b are:

a. $\frac{-11}{10}, \frac{9}{10}$ b. $\frac{-9}{10}, \frac{11}{10}$ c. $\frac{-33}{2}, 6$ d. $-6, \frac{33}{5}$

Q 17. Graphically, the pair of equations $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$ represents two lines which are: [NCERT EXEMPLAR]

a. intersecting at exactly one point
b. intersecting at exactly two points
c. coincident
d. parallel

Q 18. The pair of equations $x + 3y + 5 = 0$ and $-3x - 9y + 2 = 0$ has:

a. a unique solution
b. exactly two solutions
c. infinitely many solutions
d. no solution

Q 19. Find the value of k for which the system of equations $x + 3y = 4$ and $3x + ky + 12 = 0$ are inconsistent.

a. $k = 12$ b. $k = -12$
c. $k = 9$ b. $k = -9$

Q 20. If $x = a$ and $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 8$, then the value of ab is:

a. 15 b. 16 c. -15 d. 20

Assertion & Reason Type Questions

Directions (Q. Nos. 21-25): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 21. **Assertion (A):** $x = 2, y = 1$ is a solution of pair of equations $3x - 2y = 4$ and $2x + y = 5$.
Reason (R): A pair of values (x, y) satisfying each one of the equations in a given system of two simultaneous linear equations in x and y is called a solution of the system of equations.

Q 22. **Assertion (A):** The system of equations $x + 2y - 5 = 0$ and $2x - 6y + 9 = 0$ has infinitely many solutions.
Reason (R): The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has infinitely many solutions when $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Q 23. **Assertion (A):** Graphically, the pair of linear equations $2x - y - 5 = 0$ and $x - y - 3 = 0$ represent intersecting lines.
Reason (R): The linear equations $2x - y - 5 = 0$ and $x - y - 3 = 0$ meet the Y-axis at $(0, 3)$ and $(0, -5)$.

Q 24. **Assertion (A):** The system of linear equations $3x + 5y - 4 = 0$ and $15x + 25y - 25 = 0$ is inconsistent.
Reason (R): The pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is inconsistent if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$ [CBSE 2023]

Q 25. **Assertion (A):** If the system of equations $2x + 3y = 7$ and $2ax + (a + b)y = 28$ has infinitely many solutions, then $2a - b = 0$.
Reason (R): The system of equations $3x - 5y = 9$ and $6x - 10y = 8$ has a unique solution.

Fill in the Blanks Type Questions

Q 26. Graphically, the pair of equations $x = a$ and $y = b$ represents line are .... [NCERT EXEMPLAR]

Q 27. If a pair of linear equations is consistent, then the lines representing them are either .... or ....

Q 28. Dependent pair of linear equations is always ....

Q 29. The method in which we make the coefficient of one of the unknown variable same is .... method.

Q 30. The pair of equations $2x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has .... solution.

[NCERT EXEMPLAR]

Q 31. If $x = a$ and $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are respectively ....

Q 32. The value of c for which the pair of equations $cx + y = 3$ and $6x + 2y = 6$ will have infinitely many solutions, is ....

True/False Type Questions

Q 33. If two lines are parallel, then there is no solution.

Q 34. When two lines are coincident, then system of equations is called consistent with infinitely many solutions.

Q 35. The pair of linear equations $2px + 5y = 7$ and $6x - 5y = 11$ has a unique solution if $p \neq -3$.

Q 36. In substitution method, the value of one variable in terms of another variable and substitute it in other equation to solve the linear equations.

Q 37. The solution of the equations $3x - 5y = 4$ and $9x = 2y + 7$ is $x = -\frac{9}{13}$ and $y = \frac{5}{13}$.

Solutions

1. (d) Both substitution and elimination methods are used to solve the pair of linear equations algebraically.

2. (b) Let the two supplementary angles are θ and $(180° - \theta)$

TiP

Sum of two supplementary angles is equal to 180°.

Suppose larger angle between these two is $(180° - \theta)$.
According to question.

$$(180° - \theta) = \theta + 18°$$

$$\Rightarrow \quad \theta + \theta = 180° - 18°$$

$$\Rightarrow \quad 2\theta = 162° \Rightarrow \theta = \frac{162°}{2} = 81°$$

$\therefore$ Measure of larger $= 180° - \theta = 180° - 81° = 99°$

3. (b) Given equation of lines are,

$$y = x \quad \text{and} \quad x = 4$$

Solving these two equations, we get

$$x = y = 4$$

$\therefore$ Intersecting point of these lines *i.e.* 'P'
$=$ co-ordinates of point $P = (4, 4)$.

4. (a) Given equation of line is,

$$3x - y = 3 \quad ...(1)$$

and equation of Y-axis is,

$$x = 0 \quad ...(2)$$

Put the value of x in eq. (1), we get

$$3 \times 0 - y = 3 \Rightarrow y = -3$$

So, point of intersection is $(0, -3)$.

5. (a) Given equations are

$$32x + 33y = 34 \quad ...(1)$$

and

$$33x + 32y = 31 \quad ...(2)$$

Adding eqs. (1) and (2), we get

$$65x + 65y = 65$$

$$\Rightarrow \quad x + y = 1 \quad \text{(dividing by 65)}$$

or

$$y = 1 - x \quad ...(3)$$

Put the value of y in eq. (1), we get

$$32x + 33(1 - x) = 34$$

$$32x + 33 - 33x = 34$$

$$\Rightarrow \quad -x = 1 \Rightarrow x = -1$$

Put the value of x in eq. (3), we get

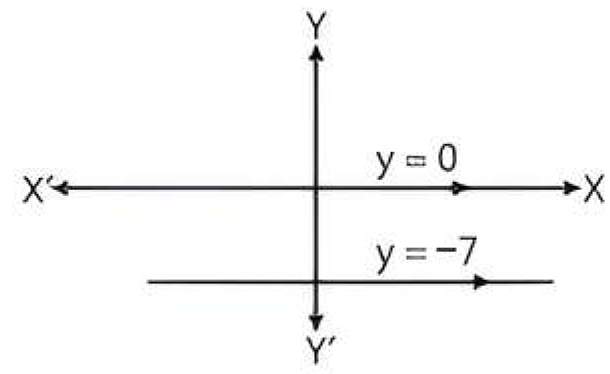
$$y = 1 - (-1) = 2$$

Hence, $x = -1, y = 2$

6. (d) Since, $y = 0$ and $y = -7$ are parallel lines, hence they have no solution.

TR!CK

Parallel lines never intersect each other.



7. (b) Given that the pair of linear equations is non-intersecting *i.e.* lines are parallel. So, the pair of linear equations is inconsistent, if it has no solution at all.

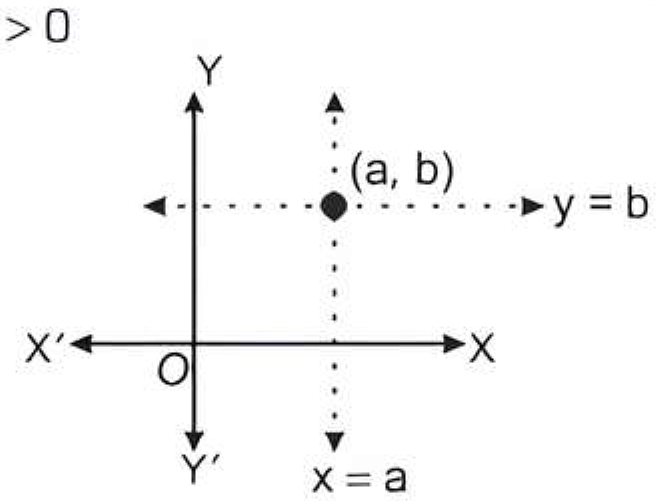
$\therefore$ Condition for no solution is:

$$\frac{a_1}{a_2} = \frac{b_1}{b_1} \neq \frac{c_1}{c_2}$$

8. (d) By graphically in every condition, if $a, b > 0$; $a, b < 0$; $a > 0, b < 0$; $a < 0, b > 0$ but $a = b \neq 0$.

The pair of equations $x = a$ and $y = b$ graphically represents lines which are intersecting at (a, b).

If $a, b > 0$



Similarly, in all cases, two lines intersect at (a, b).

9. (a) The given pair of equations is

$$2x - 3y + 4 = 0 \text{ and } 2x + y - 6 = 0$$

Subtracting second from first equation, we get

$$(2x - 3y + 4) - (2x + y - 6) = 0$$

$\Rightarrow \quad -4y+10=0 \Rightarrow y=\frac{10}{4}$

$\Rightarrow \quad y=\frac{5}{2}$

Put $y=5$ in equation $2x-3y=4$, we get

$2x-3\left(\frac{5}{2}\right)+4=0 \Rightarrow 2x-\frac{15}{2}+4=0$

$\Rightarrow \quad 2x-\frac{7}{2}=0 \Rightarrow x=\frac{7}{4}$

Hence, given pair of equations has a unique solution.

10. (c) Given, pair of linear equations are:

$x+2y-5=0$ .... (1)

and $2x-4y+6=0$ ....(2)

On comparing with the equations

$a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$

respectively, we get

$a_1=1, \quad b_1=2, c_1=-5;$
$a_2=2, b_2=-4, c_2=6.$

Now, $\frac{a_1}{a_2}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{2}{-4}=-\frac{1}{2}$

$\therefore \quad \frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

$\therefore$ Given pair of linear equations is consistent with a unique solution.

11. (c) Given pair of linear equations is,

$2x=5y+6 \Rightarrow 2x-5y-6=0$ ...(1)

and $15y=6x-18 \Rightarrow 6x-15y-18=0$ ...(2)

Comparing eqs. (1) and (2) with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ respectively, we get

$a_1=2, \quad b_1=-5, \quad c_1=-6$

and $a_2=6, \quad b_2=-15, c_2=-18$

Now, $\frac{a_1}{a_2}=\frac{2}{6}=\frac{1}{3}, \frac{b_1}{b_2}=\frac{-5}{-15}=\frac{1}{3}$ and $\frac{c_1}{c_2}=\frac{-6}{-18}=\frac{1}{3}$

$\because \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_1}$

$\therefore$ The pair of equations represent coincident lines.

12. (d) Given pair of linear equations is,

$3x-y+8=0$ ...(1)

and $6x-ry+16=0$ ...(2)

Comparing eqs. (1) and (2) with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, we get

$a_1=3, b_1=-1, c_1=8$ and $a_2=6, b_2=-r, c_2=16$

Since, the given pair of linear equations represent coincident lines *i.e.* it has infinitely many solutions.

$\therefore \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$\Rightarrow \quad \frac{3}{6}=\frac{-1}{-r}=\frac{8}{16}$

Taking $\frac{3}{6}=\frac{-1}{-r}$, we get

$r=\frac{6}{3}=2$

13. (a) Given pair of lines is

$5x+7y=3 \Rightarrow 5x+7y-3=0$

and $15x+21y=k \Rightarrow 15x+21y-k=0$

Compare these lines with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ respectively, we get

$a_1=5, \quad b_1=7, \quad c_1=-3$

and $a_2=15, \quad b_2=21, \quad c_2=-k$

Condition for coincide lines is:

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2} \Rightarrow \frac{5}{15}=\frac{7}{21}=\frac{-3}{-k}$

$\Rightarrow \quad \frac{1}{3}=\frac{1}{3}=\frac{3}{k} \Rightarrow \frac{1}{3}=\frac{3}{k}\Rightarrow k=9$

14. (b) The given pair of linear equations is

$2x+3y-6=0$ and $2ax+(a+b)y-24=0$

Here, $a_1=2, b_1=3, c_1=-6, a_2=2a, b_2=a+b, c_2=-24$

The given pair of linear equations has infinitely many solutions, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

$\Rightarrow \quad \frac{2}{2a}=\frac{3}{a+b}=\frac{-6}{-24} \Rightarrow \frac{1}{a}=\frac{3}{a+b}=\frac{1}{4}$

$\Rightarrow \quad \frac{1}{a}=\frac{3}{a+b} \Rightarrow a+b=3a \Rightarrow b=2a$

15. (b)

TR!CK

The condition for pair of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ has infinitely many solutions, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

Here, $\frac{\sin\theta}{1}=\frac{\cos\theta}{1}=\frac{1}{\sqrt{2}}$

$\Rightarrow \quad \sin\theta=\frac{1}{\sqrt{2}}, \cos\theta=\frac{1}{\sqrt{2}}$

$\therefore \quad \theta=45^\circ$

16. (c) The pair of equations is $5x-3y-9=0$

and $(a-b)x-(a+b-3)y-(a-4b)=0$

Here, $a_1=5, b_1=-3, c_1=-9, a_2=a-b$

$b_2=-(a+b-3), c_2=-(a-4b)$

For the equations to be coincident, we have

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$\Rightarrow \quad \frac{5}{a-b}=\frac{-3}{-(a+b-3)}=\frac{-9}{-(a-4b)}$

$$\Rightarrow \quad \frac{5}{a-b}=\frac{3}{a+b-3} \text{ and } \frac{3}{a+b-3}=\frac{9}{a-4b}$$

$\Rightarrow \quad 5a+5b-15=3a-3b$

and $\quad 3a-12b=9a+9b-27$

$\Rightarrow \quad 2a+8b-15=0$ ...(1)

and $\quad 6a+21b-27=0$

or $\quad 2a+7b-9=0$ ...(2)

Subtracting eq. (2) from eq. (1), we get

$$b-6=0 \Rightarrow b=6$$

Substituting this value of b in eq. (1), we get

$$2a+8\times6-15=0$$

$\Rightarrow \quad 2a+48-15=0$

$\Rightarrow \quad 2a=-33 \Rightarrow a=\frac{-33}{2}$

17. (d) Given pair of equations is,

$6x-3y+10=0$ ...(1)

and $\quad 2x-y+9=0$ ...(2)

Comparing eqs. (1) and (2) with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ respectively, we get

$a_1=6$, $b_1=-3$, $c_1=10$ and $a_2=2$, $b_2=-1$, $c_2=9$.

Now, $\frac{a_1}{a_2}=\frac{6}{2}=\frac{3}{1}$; $\frac{b_1}{b_2}=\frac{-3}{-1}=\frac{3}{1}$; $\frac{c_1}{c_2}=\frac{10}{9}$

$\because \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\therefore$ The pair of equations represents two lines which are parallel.

18. (d) The given equations are $x+3y+5=0$ and $-3x-9y+2=0$.

Here, $a_1=1, b_1=3, c_1=5, a_2=-3, b_2=-9, c_2=2$

Now, $\frac{a_1}{a_2}=\frac{-1}{3}$, $\frac{b_1}{b_2}=\frac{3}{-9}=\frac{-1}{3}$ and $\frac{c_1}{c_2}=\frac{5}{2}$

$\because \quad \frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$\therefore$ The given pair of equations has no solution.

19. (c) The given equations are

$$x+3y-4=0 \text{ and } 3x+ky+12=0$$

Here, $a_1=1, b_1=3, c_1=-4, a_2=3, b_2=k, c_2=12$

For inconsistent equations,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2} \Rightarrow \frac{1}{3}=\frac{3}{k}\neq\frac{-4}{12}$$

$\Rightarrow k=9$ and $k\neq-9$

20. (a) Since, $x=a$ and $y=b$ is the solution of given equations $x-y=2$ and $x+y=8$.

$\therefore \quad a-b=2$ ...(1)

and $\quad a+b=8$ (put $x=a$ and $y=b$) ...(2)

Adding eqs. (1) and (2), we get

$$2a=10 \Rightarrow a=5$$

Put $a=5$ in eq. (1), we get

$$5-b=2 \Rightarrow b=3$$

$\therefore \quad ab=5\times3=15$

21. (a) **Assertion (A):** The given system of equations is

$3x-2y=4$ ...(1)

and $\quad 2x+y=5$ ...(2)

Putting $x=2$ and $y=1$ in eq. (1), we get

LHS $=3\times2-2\times1=4=$ RHS

Putting $x=2$ and $y=1$ in eq. (2), we get

LHS $=2\times2+1\times1=5=$ RHS

Thus, $x=2$ and $y=1$ satisfy both the equations of the given system.

Hence, $x=2$, $y=1$ is a solution of the given pair of equations.

So, Assertion (A) is true.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

22. (d) **Assertion (A):** The given system of equations is

$x+2y-5=0$ and $2x-6y+9=0$

Here, $\frac{a_1}{a_2}=\frac{1}{2}$, $\frac{b_1}{b_2}=\frac{2}{-6}=\frac{-1}{3}$, $\frac{c_1}{c_2}=\frac{-5}{9}$

Since, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

$\therefore$ The given system of equations has a unique solution.

So, Assertion (A) is false.

Reason (R): It is true to say that the system of equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ has infinitely many solutions, when $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

Hence, Assertion (A) is false but Reason (R) is true.

23. (c) **Assertion (A):** The given system of linear equations are

$2x-y-5=0$ ...(1)

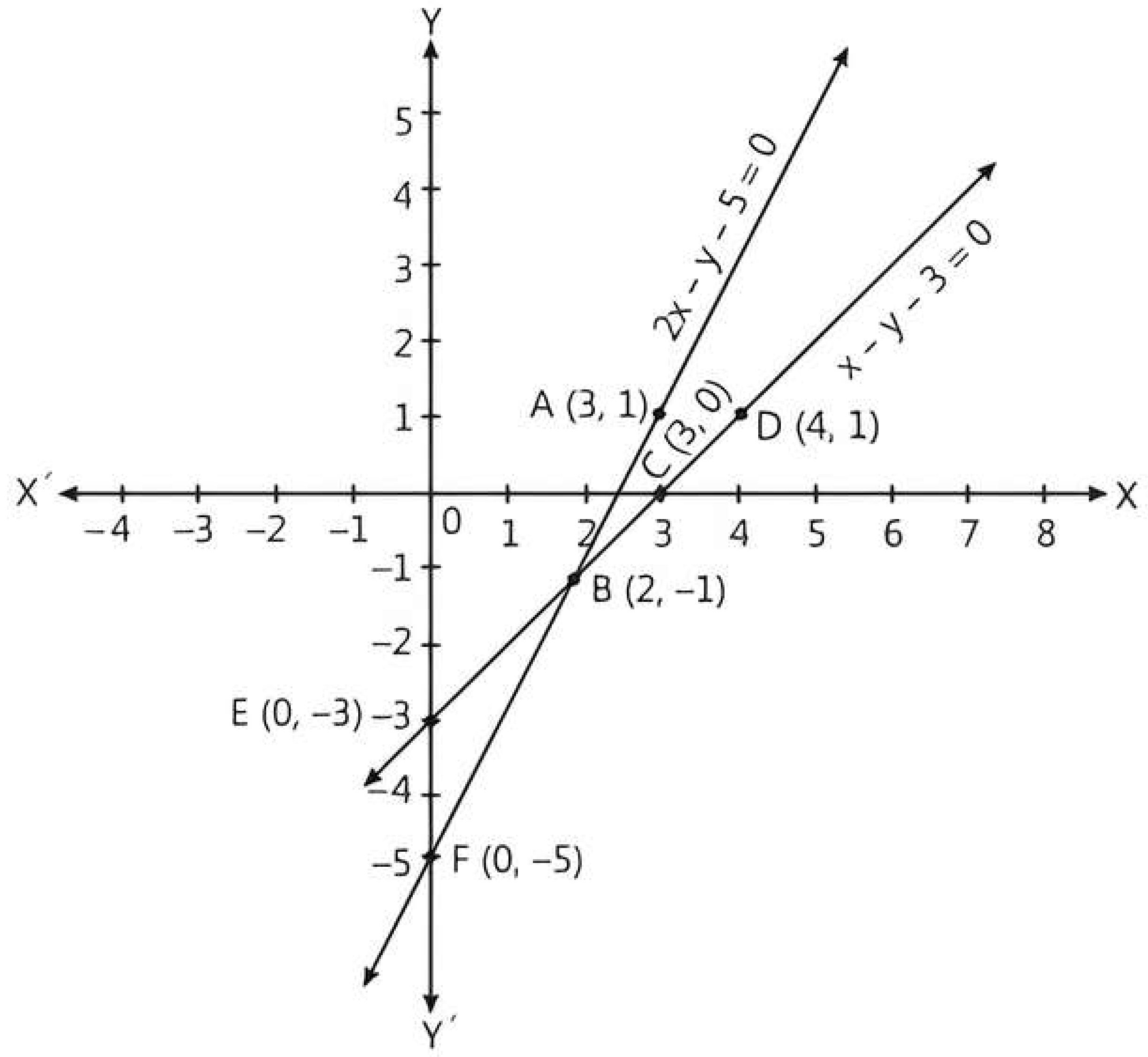
and $\quad x-y-3=0$ ...(2)

Table for eqs. (1) and (2) are given below:

x	3	2
y	1	–1

x	3	4
y	0	1

The graphical representation of the given pair of linear equations is as follows:



In the graph, we observe that the two lines intersect at the point B(2, –1).

So, $x = 2$, $y = -1$ is the required solution of the given pair of linear equations.

So, Assertion (A) is true.

Reason (R): We observe from the graph that the lines (1) and (2) meet the Y-axis at the points E(0, –3) and F(0, –5) respectively.

So, Reason (R) is false.

Hence, Assertion (A) is true but Reason (R) is false.

24. (a) **Assertion (A):** Given system of linear equations are

$$3x + 5y - 4 = 0 \quad ...(1)$$

and $$15x + 25y - 25 = 0 \quad ...(2)$$

On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ respectively, we get

$$a_1 = 3, b_1 = 5, c_1 = -4$$
$$a_2 = 15, b_2 = 25, c_2 = -25$$

Now, $\frac{a_1}{a_2} = \frac{3}{15} = \frac{1}{5}, \frac{b_1}{b_2} = \frac{5}{25} = \frac{1}{5}$ and $\frac{c_1}{c_2} = \frac{-4}{-25} = \frac{4}{25}$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

We know that, the pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is inconsistent, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of linear equations are inconsistent.

Thus, Assertion (A) is true.

Reason (R) : It is also true statement.

Hence both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

25. (c) **Assertion (A):** Given system of equations has infinitely many solutions, if

$$\frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{-28}$$

i.e., $$\frac{1}{a} = \frac{3}{a+b} = \frac{1}{4}$$

$$\therefore \quad 3a = a + b \Rightarrow 2a - b = 0$$

So, Assertion (A) is true.

Reason (R): For unique solution, condition is

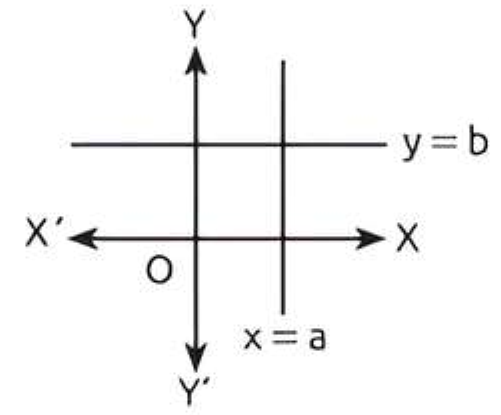
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

But here $$\frac{3}{6} = \frac{-5}{-10} \Rightarrow \frac{1}{2} = \frac{1}{2}$$

So, Reason (R) is false.

Hence, Assertion (A) is true but Reason (R) is false.

26. Graphically, the pair of equations $x = a$ and $y = b$ represents lines are intersecting.



27. intersecting, coincident

28. consistent.

29. elimination

30. Given system of equations are

$$2x + 2y + 5 = 0 \quad ...(1)$$

and $$-3x - 6y + 1 = 0 \quad ...(2)$$

Multiplying eq. (1) by 3 and then adding it with eq. (2), we get

$$6x - 3x + 15 + 1 = 0$$
$$\Rightarrow \quad 3x + 16 = 0$$
$$\Rightarrow \quad x = -\frac{16}{3}$$

Put $x = -\frac{16}{3}$ in eq. (1), we get

$$2\left(\frac{-16}{3}\right) + 2y + 5 = 0$$

$$\Rightarrow \quad 2y = -5 + \frac{32}{3} \Rightarrow y = \frac{17}{6}$$

Hence, it has unique solution.

31. Since, a and b are the solutions of the given system of equations.

$$\therefore \quad a - b = 2 \quad \text{and} \quad a + b = 4$$

Adding both equations, we get

$$2a = 6 \Rightarrow a = 3$$

Now, $$3 - b = 2 \Rightarrow b = 1$$

Hence, $a = 3, b = 1$

32. Given pair of equations is

$$cx + y - 3 = 0 \quad \text{and} \quad 6x + 2y - 6 = 0$$

Here, $a_1 = c, \; b_1 = 1, \; c_1 = -3$

and $a_2 = 6, \; b_2 = 2, \; c_2 = -6$

Now, $\frac{a_1}{a_2} = \frac{c}{6}, \frac{b_1}{b_2} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-3}{-6} = \frac{1}{2}$

TR!CK

The condition of infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\therefore \quad \frac{c}{6} = \frac{1}{2} = \frac{1}{2} \quad \Rightarrow \quad \frac{c}{6} = \frac{1}{2}$

$\Rightarrow \quad c = 3$

33. True

34. True

35. The condition for unique solution is

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\therefore \quad \frac{2p}{6} \neq \frac{5}{-5} \Rightarrow p \neq -\frac{6}{2}$

$\Rightarrow \quad p \neq -3$

So, given statement is true.

36. True

37. Given equations are

$3x - 5y = 4$ ...(1)

and $9x - 2y = 7$ ... (2)

Multiplying eq. (1) by 3 and subtracting resultant equation from eq. (2), we get

$$(9x - 2y) - (9x - 15y) = 7 - 12$$

$\Rightarrow \quad 13y = -5 \Rightarrow y = -\frac{5}{13}$

Put $y = -\frac{5}{13}$ in eq. (1), we get

$$3x - 5\left(\frac{-5}{13}\right) = 4$$

$\Rightarrow \quad 39x + 25 = 4 \times 13$

$\Rightarrow \quad 39x = 52 - 25$

$\Rightarrow \quad x = \frac{27}{39} = \frac{9}{13}$

Hence, solution is $x = \frac{9}{13}$ and $y = -\frac{5}{13}$.

So, the given statement is false.

Case Study Based Questions

Case Study 1

Sanjeev a student of class X, goes to Yamuna river with his friends. When he saw a boat in the river, then he wants to sit in boat. So his all friends are ready to sit with him. In this order, Sanjeev is sitting on a boat which upstream at a speed of 8 km/h and downstream at a speed of 16 km/h. When Sanjeev is in the boat, some questions are arises in the mind, then answer the given questions.

Based on the above information, solve the following questions:

Q 1. The speed of the boat in still water is:

a. 8 km/h b. 10 km/h

c. 12 km/h d. 14 km/h

Q 2. The speed of stream is:

a. 3 km/h b. 4 km/h

c. 5 km/h d. 6 km/h

Q 3. Which mathematical concept is used in above problem?

a. Pair of linear equations

b. Cross-multiplication method

c. Factorisation method

d. None of the above

Q 4. The direction in which the speed is maximum, is:

a. upstream

b. downstream

c. both have equal speed

d. None of the above

Q 5. The average speed of stream and boat in still water is:

a. 7 km/h

b. 10 km/h

c. 12 km/h

d. 5 km/h

Solutions

1. Let the speed of the boat in still water be x km/h and speed of the stream be y km/h.

Then, $x - y = 6$ ...(1)

$x + y = 14$ ...(2)

On adding eqs. (1) and (2), we get

$2x = 20$

$\Rightarrow \quad x = 10$

$\therefore$ Speed of the boat in still water is 10 km/h

So, option (b) is correct.

2. On putting the value of x in eq. (2), we get

$10 + y = 14$

$\Rightarrow \quad y = 14 - 10 = 4$

$\therefore$ Speed of the stream is 4 km/h.

So, option (b) is correct.

3. Pair of linear equation concept is used in above problem.

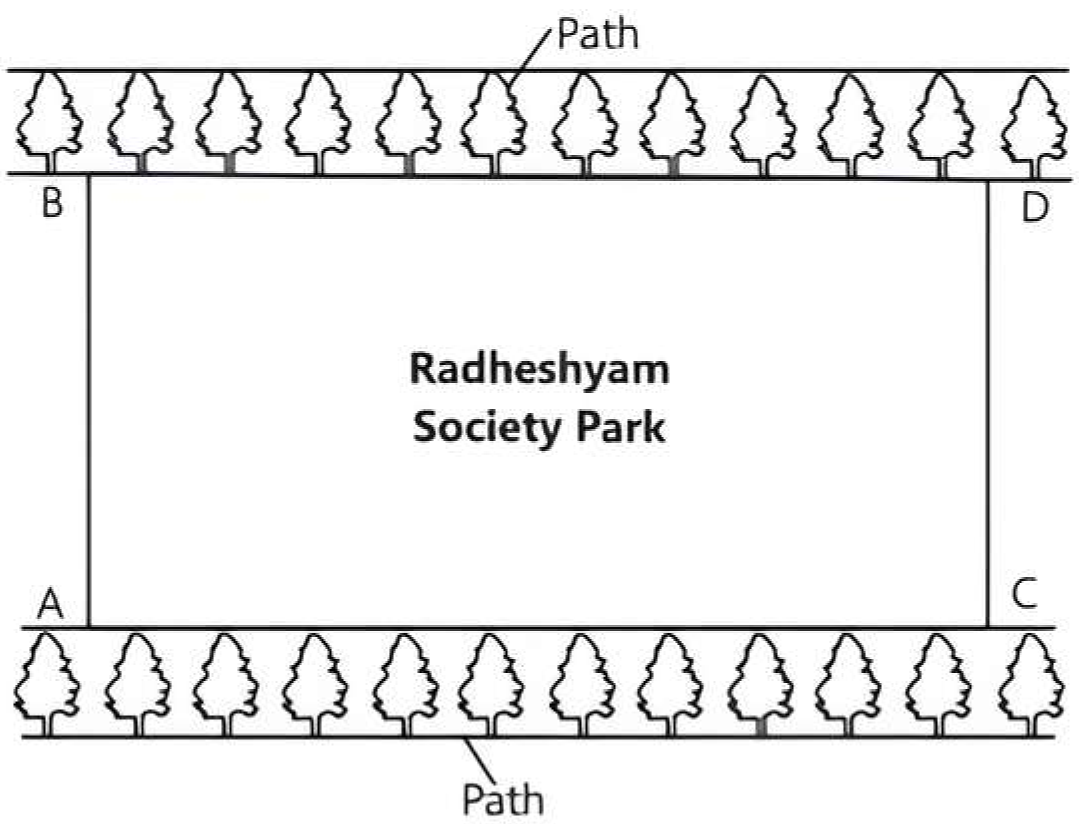
So, option (a) is correct.

4. In downstream, the speed is maximum because in downstream, the speed is $(x + y)$ km/h and in upstream, the speed is $(x - y)$ km/h.
So, option (b) is correct.

5. Average speed of stream and boat in still water
$$= \frac{y+x}{2} = \frac{4+10}{2} = \frac{14}{2} = 7 \text{ km/h}$$
So, option (a) is correct.

Case Study 2

The resident welfare association of a Radheshyam society decided to build two straight paths in their neighbourhood park such that they do not cross each other and also plant trees along the boundary lines of each path.



One of the members of association Shyam Lal suggested that the paths should be constructed represented by the two linear equations $x - 3y = 2$ and $-2x + 6y = 5$.

Based on the above information, solve the following questions:

Q 1. If the pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has infinitely solutions, then condition is:

a. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$ b. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

c. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ d. None of these

Q 2. If pair of lines are parallel, then pair of linear equations is:

a. inconsistent
b. consistent
c. consistent or inconsistent
d. None of the above

Q 3. Check whether the two paths will cross each other or not.

a. yes b. no
c. can't say d. None of these

Q 4. How many point(s) lie on the line $x - 3y = 2$?

a. one b. two
c. three d. infinitely

Q 5. If the line $2x + 6y = 5$ intersect the X-axis, then find its coordinate.

a. (–2.5, 0) b. (2.5, 0)
c. (0, 2.5) d. (0, –2.5)

Solutions

1. If the pair of equation $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has infinitely solutions, then condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
So, option (b) is correct.

2. If pair of lines are parallel, then pair of linear equation is consistent.
So, option (b) is correct.

3. Given, equation of paths are
$$x - 3y = 2 \quad ...(1)$$
and $$-2x + 6y = 5 \quad ...(2)$$
Here, $a_1 = 1, b_1 = -3, c_1 = -2$
and $a_2 = -2, b_2 = 6, c_2 = -5$
$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{-2}, \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$
$$\because \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$
So, the paths represented by the equations are parallel *i.e.*, do not intersect each other.
So, option (b) is correct.

4. Infinitely point lies on the line $x - 3y = 2$.
So, option (d) is correct.

5. The y-coordinate on X-axis is zero.
Put $y = 0$ in $2x + 6y = 5$, we get
$$2x + 6(0) = 5$$
$$\Rightarrow \quad x = \frac{5}{2} = 2.5$$
Hence, the coordinates of X-axis is (2.5, 0).
So, option (b) is correct.

Case Study 3

Akhila went to a fair in her village. She wanted to enjoy rides on the giant wheel and play hoopla (a game in which you throw a ring on the items kept in a stall and if the ring covers any object completely you get it). The number of times she played hoopla is half the number of times she rides the giant wheel. If each ride costs ₹ 3 and a game of hoopla costs ₹ 4 and she spent ₹ 20 in the fair.

Based on the above information, solve the following questions:

Q 1. Write the representation of given statement algebraically.

Q 2. Find the intersection point of two lines.

Q 3. Find the intersection points of the line $x - 2y = 0$ on X and Y-axes.

OR

Intersection points of the line $3x + 4y = 20$ on X and Y-axes.

Solutions

1. Let x and y be the number of rides on the giant wheel and number of hoopla respectively played by Akhila.

   Then, according to the given condition,

$$y = \frac{x}{2} \text{ and } 3x + 4y = 20$$

$\therefore$ The given situation can be algebraically represented by the following pair of the linear equations are

$$x - 2y = 0 \quad \text{...(1)}$$

and $$3x + 4y = 20 \quad \text{...(2)}$$

2. Put $x = 2y$ in eq. (2), we get

$$3(2y) + 4y = 20$$

$\Rightarrow$ $$10y = 20$$

$\Rightarrow$ $$y = 2$$

$\therefore$ $$x = 2 \times 2 = 4$$

Hence, intersection point of two lines is (4, 2).

3. Table for equation $x - 2y = 0$ is:

x	0
$y = \frac{x}{2}$	0
Points	(0, 0)

I.e., the lines passes through the origin.

Or

Table for equation $3x + 4y = 20$ is:

x	0	20/3
$y = \frac{20-3x}{4}$	0	0
Points	(0, 5)	(20/3, 0)

I.e., the intersection points of the line on X and Y-axes are $\left(\frac{20}{3}, 0\right)$ and (0, 5).

Case Study 4

The residents of a housing society at Jaipur decided to build a rectangular garden to beautify the garden.

One of the members of the society made some calculations and informed that if the length of the rectangular garden is increased by 2m and the breadth reduced by 2 m, the area gets reduced by 12 sq. m. However, when the length is decreased by 1 m and breadth increased by 3 m, the area of the rectangular garden is increased by 21 sq. m.

Based on the above information, solve the following questions:

Q 1. Find the coordinates of the points on X-axis, where the two lines, plotted on a graph paper intersect the X-axis.

Q 2. Find the value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$ has no solution.

Q 3. Find the dimensions of the rectangle.

OR

If the graphs of the equations in the given situation are plotted on the same graph paper, then lines will intersect. Check whether given statement is True/False.

Solutions

1. The points where the two lines will intersect the X-axis can be found by putting $y = 0$ in both the equations, as the y-coordinate of all points lying on the X-axis is zero.

Putting $y = 0$ in the equations $x - y = 4$ and $3x - y = 24$, we get $x = 4$ and $x = 8$, respectively.

Therefore, the points are (4, 0) and (8, 0).

2. For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{1}{2} = \frac{1}{k} \neq \frac{4}{3}$$

$$\Rightarrow \quad k = 2$$

3. Let the length and breadth of the rectangular garden be denoted by x m and y m respectively. The area of the rectangular garden = xy sq. m.

According to the question,

$$(x + 2)(y - 2) = xy - 12$$

Simplifying the above equations, we get

$$xy - 2x + 2y - 4 = xy - 12$$

$$\Rightarrow \quad -2x + 2y = -8$$

$$\Rightarrow \quad x - y = 4 \quad ...(1)$$

And $\quad (x - 1)(y + 3) = xy + 21$

$$xy + 3x - y - 3 = xy + 21$$

$$\Rightarrow \quad 3x - y = 24 \quad ...(2)$$

Let us now solve the eqs. (1) and (2) by the method of substitution.

From eq. (1), $\quad x = y + 4 \quad$ ...(3)

Substituting in eq. (2),

$$3(y + 4) - y = 24$$

$$\Rightarrow \quad 3y + 12 - y = 24$$

$$\Rightarrow \quad 2y = 12 \Rightarrow y = 6$$

Substituting $y = 6$ in eq. (3), $x = 6 + 4 = 10$

Therefore, length = 10 m and breadth = 6 m.

COMMON ERROR

Some students are not able to frame this word problem into equations. So adequate practice should be needed on solving such type of problem.

OR

We can find whether lines will be intersecting, coincident or parallel, by calculating the ratios of the coefficients of the pair of linear equations.

As the two equations are given by

$$x - y = 4$$

and $$3x - y = 24$$

Let us calculate the ratios of their coefficients

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{-1} = 1, \frac{c_1}{c_2} = \frac{4}{24} = \frac{1}{6}$$

As, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the two lines will intersect at a point.

Hence, given statement is true.

Case Study 5

Gagan went to a fare. He ate several rural delicacies such as jalebis, chaat etc. He also wanted to play the ring game in which a ring is thrown on the items displayed on the table and the balloon shooting game.

The cost of three balloon shooting games exceeds the cost of four ring games by ₹ 4. Also, the total cost of three balloon shooting games and four ring games is ₹ 20.

Based on the above information, solve the following questions:

Q 1. Taking the cost of one ring game to be ₹ x and that of one balloon game as ₹ y, find the pair of linear equations describing the given statement.

Q 2. Find the total cost of five ring games and eight balloon games.

Q 3. Find the cost of one balloon game.

OR

Cost of which game is more and by how much?

Solutions

1. Given, the cost of one ring game = ₹ x and cost of one balloon game = ₹ y.

According to the question,

$$3y = 4x + 4 \quad \text{or} \quad 4x - 3y = -4 \quad ...(1)$$

and $$4x + 3y = 20 \quad ...(2)$$

2. Total cost of five ring games and eight balloon games = $5x + 8y$

$$= 5 \times 2 + 8 \times 4$$

$$= 10 + 32$$

$$= ₹ 42.$$

3. Solving the equations $4x - 3y = -4$ and $4x + 3y = 20$ by the method of substitution.

From eq. (1), $\quad 4x = 3y - 4 \quad$ ...(3)

Substituting the value of $4x$ in eq. (2),

$$(3y - 4) + 3y = 20$$

$$\Rightarrow \quad 6y - 4 = 20$$

$$\Rightarrow \quad 6y = 24$$

$$\Rightarrow \quad y = 4$$

∴ Cost of one balloon game = ₹ 4.

OR

Now, substituting $y = 4$ in eq. (3),

$$4x = 3 \times 4 - 4 = 8 \Rightarrow x = 2$$

Therefore, cost of one ring game = ₹ 2

Thus cost of one balloon game is more and by ₹ (4 – 2) = ₹ 2.

Very Short Answer Type Questions

Q 1. Solve for x and y : $x + y = 6, 2x - 3y = 4$ [CBSE 2023]

Q 2. How many solutions does the pair of equations $y = 0$ and $y = -5$ have?

Q 3. If $47x + 31y = 18$ and $31x + 47y = 60$, then find the value of $x + y$.

Q 4. If $49x + 51y = 499, 51x + 49y = 501$, then find the values of x and y. [CBSE SQP 2022-23]

Q 5. Find the conditions to be satisfied by coefficients for which the following pair of equations $ax + by + c = 0,\ dx + ey + f = 0$ represent coincident lines.

Q 6. Given the pair of equations $ax + (a - 1)\ y = 1$ and $(a + 1)x - ay = 1$. For which one of the following values of a, there is no common solution of the given pair of equations?

Q 7. Find the values of k for which the system of equations $x + ky = 0, 2x - y = 0$ has unique solution?

Q 8. If $ax + by = a^2 - b^2$ and $bx + ay = 0$, find the value of $(x + y)$.

Q 9. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then find the value of k. [NCERT EXEMPLAR; V. Imp.]

Q 10. Find the value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions. [NCERT EXEMPLAR]

Q 11. Find the larger of two complementary angles exceeds the smaller by 16 degrees. Find them.

Q 12. Find the value of k for which the system of linear equations $x + 2y = 3$ and $5x + ky + 7 = 0$ is inconsistent. [CBSE 2020]

Q 13. Find whether the following pair of linear equations is consistent or inconsistent: [CBSE SQP 2023-24]

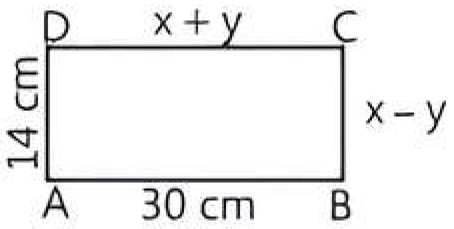
$$3x + 2y = 8$$
$$6x - 4y = 9$$

Short Answer Type-I Questions

Q 1. Find out whether the following pair of linear equations are consistent or inconsistent: [CBSE 2023]

$$5x - 3y = 11, -10x + 6y = 22$$

Q 2. In the figure, ABCD is a rectangle. Find the values of x and y. [CBSE 2018]



Q 3. Find whether the lines representing the following pair of linear equations intersect at a point or parallel or coincident:

$2x - 3y + 6 = 0$ and $4x - 5y + 2 = 0$ [CBSE 2016]

Q 4. Find the value(s) of k for which the pair of equations $kx + 2y = 3$ and $3x + 6y = 10$ has a unique solution. [CBSE 2019]

Q 5. For what value of k, does the system of linear equations $2x + 3y = 7$ and $(k - 1)x + (k + 2)y = 3k$ have an infinite number of solutions? [CBSE 2019]

Q 6. Find the value of k for which the following pair of linear equations have infinitely many solutions,

$2x + 3y = 7, (k + 1)\ x + (2k - 1)y = 4k + 1.$ [CBSE 2019]

Q 7. Find the value of k for which the system of equations $3x + 5y = 0, kx + 10y = 0$ has no solution. [CBSE 2017]

Q 8. The coach of a cricket team buys 4 bats and 1 ball for ₹ 2050. Later, she buys 3 bats and 2 balls for ₹ 1600. Find the cost of each bat and each ball.

Q 9. The sum of two numbers is 85. If the larger number is 5 more than four times of the smaller number, find the numbers.

Q 10. I am three times old as my son. Five years later, I shall be two and a half times as old as my son. How old am I and how old is my son?

Q 11. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed the charge and the charge for each extra day. [CBSE SQP 2023-24]

Short Answer Type-II Questions

Q 1. If $2x + y = 23$ and $4x - y = 19$, find the values of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$. [CBSE 2019]

Q 2. Solve:

$\frac{x}{a} + \frac{y}{b} = a + b; \frac{x}{a^2} + \frac{y}{b^2} = 2, a, b \neq 0$ [CBSE 2017]

Q 3. If the system of linear equations $2x + 3y = 7$ and $2ax + (a + b)\ y = 28$ have infinite number of solutions, then find the values of a and b. [CBSE 2023]

Q 4. Find the two numbers whose sum is 75 and difference is 15.

Q 5. A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When student A takes food for 25 days she has to pay ₹ 4500 as hostel charges whereas student B, who takes food for 30 days, pays ₹ 5200 as hostel charges. Find the fixed charges and the cost of food per day. [CBSE 2019]

Q 6. There are some students in two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But, if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in both the halls. [NCERT EXEMPLAR]

Q 7. A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction. [CBSE 2019]

Q 8. A father's age is three times the sum of the ages of his two children. After 5 years, his age will be two times the sum of their ages. Find the present age of father. [CBSE 2019]

Q 9. A person can row a boat at the rate of 5km/h in still water. He takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream. [NCERT EXEMPLAR; CBSE 2016]

Long Answer Type Questions

Q 1. Solve the following pair of linear equations graphically: $6x - y + 4 = 0$ and $2x - 5y = 8$. Shade the region bounded by the lines and Y-axis. [CBSE 2016]

Q 2. For which values of a and b will the following pair of linear equations have an infinite number of solutions?

$$2x + 3y = 7$$
$$(a - b)x + (a + b)y = 3a + b - 2$$

Q 3. Places A and B are 160 km apart on a highway. One car starts from A and another from B at the same time. If they travel in the same direction, they meet in 8 hours. But if they travel towards each other, they meet in 2 hours. Find the speed of each car.

Solutions

Very Short Answer Type Questions

1. Given equations are

$$x + y = 6 \quad ...(1)$$

and $$2x - 3y = 4 \quad ...(2)$$

Multiplying eq. (1) by 3 and adding eqs (1) and (2), we get

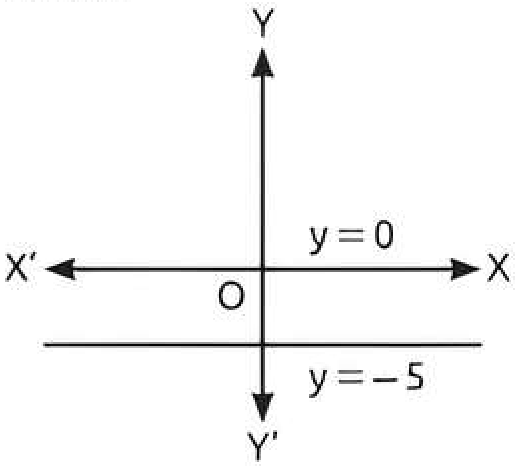
$$3(x + y) + (2x - 3y) = 18 + 4$$
$$\Rightarrow 3x + 3y + 2x - 3y = 22$$
$$\Rightarrow 5x = 22 \Rightarrow x = \frac{22}{5}$$

Put the value of x in eq. (1), we get

$$\frac{22}{5} + y = 6 \Rightarrow y = 6 - \frac{22}{5} = \frac{30-22}{5} = \frac{8}{5}$$

Hence, $x = \frac{22}{5}$ and $y = \frac{8}{5}$

2. Since, $y = 0$ and $y = -5$ are parallel lines, hence they have no solution.



TR!CK

Parallel lines never intersect each other.

3. Given,

$$47x + 31y = 18 \quad ...(1)$$

and $$31x + 47y = 60 \quad ...(2)$$

Adding eqs. (1) and (2), we get

$$78x + 78y = 78$$
$$\Rightarrow x + y = 1 \quad \text{(divide both sides by 78)}$$

COMMON ERR(!)R

Students do error in simplifying these type of equations. They solve these equations by elimination method.

4. Given equations are

$$49x + 51y = 499 \quad ...(1)$$

and $$51x + 49y = 501 \quad ...(2)$$

Adding eqs. (1) and (2), we get

$$100x + 100y = 1000$$
$$\Rightarrow x + y = 10 \quad ...(3)$$

Subtracting eq. (1) from eq. (2), we get

$$2x - 2y = 2$$
$$\Rightarrow x - y = 1 \quad ...(4)$$

Adding eqs. (3) and (4), we get

$$2x = 11 \Rightarrow x = \frac{11}{2}$$

Put $x = \frac{11}{2}$ in eq. (3), we get

$$\frac{11}{2} + y = 10$$
$$\Rightarrow y = 10 - \frac{11}{2} \Rightarrow y = \frac{9}{2}$$

5. The given pair of equations is

$$ax + by + c = 0 \text{ and } dx + ey + f = 0$$

For coincident lines,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2} \Rightarrow \frac{a}{d}=\frac{b}{e}=\frac{c}{f}$$

$\Rightarrow$ $ae = bd$ and $bf = ce$

6. The condition for there is no common point, is

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{a}{a+1}=\frac{a-1}{-a}\neq 1$$

$\Rightarrow$ $-a^2 = a^2-1$ $\Rightarrow$ $2a^2 = 1$

$\Rightarrow$ $a^2=\frac{1}{2}$ $\Rightarrow$ $a=\pm\frac{1}{\sqrt{2}}$

7. Given, $x+ky=0, 2x-y=0$

Here, $a_1 = 1, b_1 = k, c_1 = 0,$

and $a_2 = 2, b_2 = -1, c_2 = 0$

$\because$ System of equations has unique solution.

$\therefore$ $\frac{a_1}{b_2}\neq\frac{b_1}{b_2} \Rightarrow \frac{1}{2}\neq\frac{k}{-1} \Rightarrow k\neq-\frac{1}{2}$

8. Given equations are

$ax+by=a^2-b^2$ ...(1)

and $bx+ay=0$ ...(2)

On adding both equations, we get

$(ax+by)+(bx+ay)=a^2-b^2$

$\Rightarrow$ $(a+b)x+(a+b)y=a^2-b^2$

$\Rightarrow$ $(a+b)(x+y)=(a+b)(a-b)$

Hence, $x+y=a-b$.

COMMON ERROR

Students do error in simplifying these type of equations.

9. Since, the given lines are parallel, so it has no solution.

TRICK

Condition for no solution,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$\therefore$ $\frac{3}{2}=\frac{2k}{5}\neq\frac{-2}{1}$

$\Rightarrow$ $4k=15$

Hence, $k=\frac{15}{4}$

10. Given equations are,

$cx-y-2=0$ ...(1)

and $6x-2y-3=0$ ...(2)

Here, $a_1=c, b_1=-1, c_1=-2$

$a_2=6, b_2=-2, c_2=-3$

Condition for infinitely many solutions is

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2} \Rightarrow \frac{c}{6}=\frac{-1}{-2}=\frac{-2}{-3}$$

$\Rightarrow$ $\frac{c}{6}=\frac{1}{2}$ and $\frac{c}{6}=\frac{2}{3}$

$\Rightarrow$ $c=\frac{6}{2}=3$ and $c=\frac{6\times 2}{3}=4$

Here, $c=3$ and 4, which are not possible.

Since, these values are not satisfying the given condition, so no value of c will exist.

COMMON ERROR

Sometimes students do not crosschecking the values of c with the given pair of equations. Finally get the wrong answer.

11. Let the two angles be x and y where $x>y$.

According to the question,

$x+y=90°$ ...(1)

and $x-y=16°$ ...(2)

On adding eqs. (1) and (2), we get

$2x=106° \Rightarrow x=53°$

Substituting the value of x in eq. (1), we get

$y=90°-53°=37°$

12. Given, $x+2y=3$ and $5x+ky+7=0$

Here, $a_1=1, b_1=2, c_1=-3$

and $a_2=5, b_2=k, c_2=7$

Condition of inconsistency is

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2} \Rightarrow \frac{1}{5}=\frac{2}{k} \Rightarrow k=10$$

13. Given pair of linear equations are:

$3x+2y=8 \Rightarrow 3x+2y-8=0$ ...(1)

and $6x-4y=9 \Rightarrow 6x-4y-9=0$ ...(2)

On comparing with standard equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ respectively.

We get

$a_1=3, b_1=2, c_1=-8;$

$a_2=6, b_2=-4, c_2=-9$

Now, $\frac{a_1}{a_2}=\frac{3}{6}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{2}{-4}=-\frac{1}{2}$

$\because$ $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

$\therefore$ The given pair of linear equations are consistent with unique solution .

Short Answer Type-I Questions

1. Given pair of linear equations are:

$5x-3y=11 \Rightarrow 5x-3y-11=0$ ...(1)

$-10x+6y=22 \Rightarrow -10x+6y-22=0$ ...(2)

On comparing with standard equations

$a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ respectively.

We get $a_1=5, b_1=3, c_1=-11$

$a_2=-10, b_2=6, c_2=-22$

Now, $\frac{a_1}{a_2}=\frac{5}{-10}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2}$

$\because$ $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

$\therefore$ The given pair of linear equations are consistent with unique solution.

2. Since, $ABCD$ is a rectangle.

TR!CK

Opposite sides of a rectangle are equal.

So, $AB = DC$

$30 = x + y$ ...(1)

and $AD = BC$

$14 = x - y$ ...(2)

On adding eqs. (1) and (2), we get

$(x + y) + (x - y) = 30 + 14$

$\Rightarrow 2x = 44$

$\Rightarrow x = \frac{44}{2} \Rightarrow x = 22$

On putting the value of 'x' in eq. (1), we get

$30 = 22 + y$

$\Rightarrow y = 30 - 22 = 8$

Hence, $x = 22$ and $y = 8$.

3. Given equations are,

$2x - 3y + 6 = 0$ ...(1)

and $4x - 5y + 2 = 0$ ...(2)

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-5} = \frac{3}{5}$

Since, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, given pair of linear equations intersect at one point.

4. Given equations are,

$kx + 2y = 3$

and $3x + 6y = 10$

Here, $a_1 = k, b_1 = 2, c_1 = -3$

and $a_2 = 3, b_2 = 6, c_2 = -10$

For unique solution,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{3} \neq \frac{2}{6} \Rightarrow k \neq 1$

Hence, the given system of equations will have a unique solution for all real values of k other than 1.

5. Given system of linear equations are

$2x + 3y - 7 = 0$

and $(k - 1)x + (k + 2)y - 3k = 0$

Here $a_1 = 2, b_1 = 3, c_1 = -7$

and $a_2 = k - 1, b_2 = k + 2, c_2 = -3k$

Condition for infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \frac{2}{k-1} = \frac{3}{k+2} = \frac{-7}{-3k}$$

Consider Ist and IInd terms,

$\frac{2}{k-1} = \frac{3}{k+2} \Rightarrow 2k + 4 = 3k - 3$

$\Rightarrow 3k - 2k = 4 + 3 \Rightarrow k = 7$

Hence, given system of equations gives the infinite solutions, when $k = 7$.

6. Given pair of linear equations are

$2x + 3y = 7, (k + 1)x + (2k - 1)y = 4k + 1$

Here, $a_1 = 2, b_1 = 3, c_1 = -7$

and $a_2 = (k + 1), b_2 = (2k - 1), c_2 = -(4k + 1)$

Condition for infinitely many solutions is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \frac{2}{k+1} = \frac{3}{2k-1} = \frac{-7}{-(4k+1)}$$

Consider Ist and IInd terms.

$$\frac{2}{k+1} = \frac{3}{2k-1}$$

$\Rightarrow 4k - 2 = 3k + 3 \Rightarrow 4k - 3k = 3 + 2 \Rightarrow k = 5$

Consider IInd and IIIrd terms.

$\frac{3}{2k-1} = \frac{-7}{-(4k+1)} \Rightarrow 12k + 3 = 14k - 7$

$\Rightarrow 14k - 12k = 3 + 7 \Rightarrow 2k = 10 \Rightarrow k = 5$

Hence, $k = 5$

7. Given equations are

$3x + 5y = 0$ ...(1)

and $kx + 10y = 0$ ...(2)

Here, $a_1 = 3, b_1 = 5, c_1 = 0$

and $a_2 = k, b_2 = 10, c_2 = 0$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow \frac{3}{k} = \frac{5}{10} \Rightarrow k = \frac{30}{5} = 6$

Hence, $k = 6$.

8. Let the cost of one bat be ₹x.

Let the cost of one ball be ₹y

First condition:

4 bats and 1 ball = ₹ 2050

i.e., $4x + y = 2050$ ... (1)

Second condition:

3 bats and 2 balls = ₹ 1600

i.e., $3x + 2y = 1600$ ...(2)

Multiply by 2 in eq. (1) and subtract from eq. (2), we get

$(3x + 2y) - 2(4x + y) = 1600 - 2 \times 2050$

$3x + 2y - 8x - 2y = 1600 - 4100$

$\Rightarrow -5x = -2500$

$\therefore x = \frac{2500}{5} = 500$

Put the value of x in eq. (1), we get

$4 \times 500 + y = 2050$

$\Rightarrow y = 2050 - 2000 = 50$

So, cost of each bat and each ball are ₹ 500 and ₹ 50 respectively.

9. Let the two numbers be x and y such that $x > y$.

Given, sum of two numbers = 85

and larger number = 4 × smaller number + 5.

$\therefore \quad x + y = 85 \quad ...(1)$

and $\quad x = 4y + 5 \quad ...(2)$

Put $x = 4y + 5$ in eq. (1), we get

$4y + 5 + y = 85$

$\Rightarrow \quad 5y = 80 \Rightarrow y = 16$

Put $y = 16$ in eq. (1), we get

$x + 16 = 85$

$\Rightarrow \quad x = 69$

Hence, numbers are 16 and 69.

10. Let my age be x years and my son's age be y years. Then,

$x = 3y \quad ...(1)$

Five years later, my age will be $(x + 5)$ years and my son's age will be $(y + 5)$ years.

$\therefore \quad x + 5 = 2\frac{1}{2}(y + 5)$

$\Rightarrow \quad x + 5 = \frac{5}{2}(y + 5)$

$\Rightarrow \quad 2x + 10 = 5y + 25$

$\Rightarrow \quad 2x - 5y = 15$

$\Rightarrow \quad 2(3y) - 5y = 15 \quad$ [from eq. (1)]

$\Rightarrow \quad 6y - 5y = 15$

$\Rightarrow \quad y = 15$

Put $y = 15$ in eq. (1), we get

$x = 3 \times 15$

$\Rightarrow \quad x = 45$

Hence, my present age and my son's present age are 45 years and 15 years respectively.

11. Let the fixed charge for first 3 days be ₹ x and additional charge after 3 days be ₹y.

Now according to given condition, we have

$x + 4y = 27 \quad ...(1)$

and $\quad x + 2y = 21 \quad ...(2)$

Subtracting eq. (2) from eq. (1), we get

$(x + 4y)\ (x + 2y) = 27 - 21$

$2y = 6 \Rightarrow y = \frac{6}{2} = 3$

Put the value of y in eq (2),

we get

$x + 2 \times 3 = 21$

$\Rightarrow \quad x = 21 - 6 = 15$

Hence, Fixed charge = ₹ 15 and additional charge per day = ₹ 3.

12. Let a factor of $f(x) = 2x^3 + ax^2 + 2bx + 1$ be $(x + 1)$.

$\therefore \quad f(-1) = 0$

$\Rightarrow \quad 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$

$\Rightarrow \quad -2 + a - 2b + 1 = 0$

$\Rightarrow \quad a - 2b = 1 \quad ...(1)$

and $\quad 2a - 3b = 4$ (given) ...(2)

Multiplying eq. (1) by 2 and then subtract it from eq. (2), we get

$(2a - 3b) - (2a - 4b) = 4 - 2$

$\Rightarrow \quad 2a - 3b - 2a + 4b = 2 \Rightarrow b = 2$

Put the value of b in eq. (1), we get

$a - 2(2) = 1 \Rightarrow a - 4 = 1$

$\Rightarrow \quad a = 5$

Therefore, $a = 5$ and $b = 2$.

Short Answer Type-II Questions

1. Given linear equations are

$2x + y = 23 \quad ... (1)$

and $\quad 4x - y = 19 \quad ... (2)$

Adding eqs. (1) and (2),

$$\begin{array}{r} 2x + y = 23 \\ \underline{4x - y = 19} \\ 6x = 42 \end{array}$$

$\therefore \quad x = \frac{42}{6} = 7$

Now, put the value of x in eq. (1), we get

$y = 23 - 2x = 23 - 2 \times 7 = 23 - 14 = 9$

So, $5y - 2x = 5 \times 9 - 2 \times 7 = 45 - 14 = 31$

and $\frac{y}{x} - 2 = \frac{9}{7} - 2 = \frac{9 - 14}{7} = \frac{-5}{7}$

2. Given equations are

$\frac{x}{a} + \frac{y}{b} = a + b \quad ...(1)$

and $\quad \frac{x}{a^2} + \frac{y}{b^2} = 2 \quad ...(2)$

On multiplying eq. (1) by $\frac{1}{b}$ and subtracting from eq. (2), we get

$\left(\frac{x}{a^2} + \frac{y}{b^2}\right) - \left(\frac{x}{ab} + \frac{y}{b^2}\right) = 2 - \left(\frac{a + b}{b}\right)$

$\Rightarrow \quad x\left\{\frac{1}{a^2} - \frac{1}{ab}\right\} = 2 - \frac{a}{b} - 1$

$\Rightarrow \quad \frac{x}{a}\left\{\frac{1}{a} - \frac{1}{b}\right\} = 1 - \frac{a}{b}$

$\Rightarrow \quad \frac{x}{a}\left(\frac{b - a}{ab}\right) = \frac{b - a}{b} \Rightarrow x = \frac{(b - a)}{b} \times \frac{ab}{(b - a)} \times a$

$\therefore \quad x = a^2$

On putting the value of 'x' in eq. (1), we get

$\frac{a^2}{a} + \frac{y}{b} = a + b \quad \Rightarrow \quad a + \frac{y}{b} = a + b$

$\Rightarrow \quad \frac{y}{b} = b \quad \Rightarrow \quad y = b^2$

Hence, $x = a^2$ and $y = b^2$.

3. Given system of linear equations is:

$2x + 3y = 7 \Rightarrow 2x + 3y - 7 = 0 \quad ...(1)$

$2ax + (a + b)y = 28 \Rightarrow 2ax + (a + b)\ y - 28 = 0 \quad ...(2)$

Compare it with $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ respectively, we get

$a_1 = 2,\ b_1 = 3,\ c_1 = -7$

and $\quad a_2 = 2a,\ b_2 = (a + b),\ c_2 = -28$

Since, the given system of linear equations have infinite number of solutions.

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{2}{2a} = \frac{3}{a + b} = \frac{-7}{-28}$

Taking Ist and IInd terms, $\frac{2}{2a} = \frac{3}{a+b} \Rightarrow a + b = 3a$

$$\Rightarrow \quad b = 2a \quad ...(1)$$

Taking Ist and IIIrd terms, $\frac{2}{2a} = \frac{-7}{-28} \Rightarrow \frac{1}{a} = \frac{1}{4} \Rightarrow a = 4$

Put the value of 'a' in eq. (1), we get

$$b = 2 \times 4 = 8$$

Hence, $a = 4$ and $b = 8$.

4. Let the two numbers be x and y.

According to the question,

$$x + y = 75 \quad ...(1)$$

and $$x - y = \pm 15 \quad ...(2)$$

TiP

Take ± sign with resultant quantity of difference of two numbers.

Taking positive sign:

On adding eqs. (1) and (2), we get

and $(x + y) + (x - y) = 75 + 15$

$$\Rightarrow \quad 2x = 90$$

$$\Rightarrow \quad x = 45$$

On putting the value of 'x' in eq. (1), we get

$$45 + y = 75$$

$$\Rightarrow \quad y = 75 - 45 = 30$$

$$\therefore \quad x = 45 \text{ and } y = 30$$

Taking negative sign:

On adding eqs. (1) and (2), we get

$$(x + y) + (x - y) = 75 - 15$$

$$\Rightarrow \quad 2x = 60$$

$$\Rightarrow \quad x = 30$$

On putting the value of x in eq. (1), we get

$$30 + y = 75$$

$$\Rightarrow \quad y = 75 - 30 = 45$$

$$\therefore \quad x = 30 \text{ and } y = 45$$

Hence, required numbers are (45, 30) or (30, 45).

5. Let x be the fixed charge of the food and y be the cost of food per day.

According to the given conditions,

$$x + 25y = 4500 \quad ...(1)$$

and $$x + 30y = 5200 \quad ...(2)$$

On subtracting eq. (1) from eq. (2), we get

$$\begin{array}{r} x + 30y = 5200 \\ x + 25y = 4500 \\ - \quad - \quad\quad - \\ \hline 5y = 700 \end{array}$$

$$\Rightarrow \quad y = 140$$

On substituting $y = 140$ in eq. (1), we get

$$x + 25 \times 140 = 4500$$

$$\Rightarrow \quad x = 4500 - 3{,}500$$

$$\Rightarrow \quad x = 1000$$

Hence, fixed charges is ₹ 1000 and cost of food per day is ₹ 140.

COMMON ERR(!)R

Some students frame this word problem into wrong equation, lead to get incorrect answer. So, adequate practice should be needed on solving such type of problem.

6. Let the number of students in halls A and B be x and y respectively.

According to first condition,

$$x - 10 = y + 10$$

$$\Rightarrow \quad x - y = 10 + 10 = 20 \quad ...(1)$$

According to second condition,

$$x + 20 = 2(y - 20)$$

$$\Rightarrow \quad x + 20 = 2y - 40$$

$$\Rightarrow \quad x - 2y = -40 - 20 = -60 \quad ...(2)$$

On subtracting eq. (2) from eq. (1), we get

$$\begin{array}{r} x - y = 20 \\ x - 2y = -60 \\ - \quad + \quad\quad + \\ \hline y = 80 \end{array}$$

On substituting the value of 'y' in eq. (1), we get

$$x - 80 = 20$$

$$\Rightarrow \quad x = 20 + 80 = 100$$

Hence, number of students in hall A and B are 100 and 80 respectively.

COMMON ERR(!)R

Some candidates frame this word problem into wrong equation, lead to get incorrect answer.

7. Let fraction be $\frac{x}{y}$, where $y \neq 0$

According to the given condition,

When 2 is subtracted from numerator then fraction becomes $\frac{1}{3}$.

i.e. $$\frac{x-2}{y} = \frac{1}{3} \Rightarrow 3x - 6 = y \quad ...(1)$$

and when 1 is subtracted from denominator then fraction becomes $\frac{1}{2}$.

i.e. $$\frac{x}{y-1} = \frac{1}{2}$$

$$\Rightarrow \quad 2x = y - 1 \quad ...(2)$$

From eq. (1), put the value of y in eq. (2), we get

$$2x = (3x - 6) - 1$$

$$\Rightarrow \quad 2x = 3x - 7$$

$$\Rightarrow \quad 3x - 2x = 7$$

$$\Rightarrow \quad x = 7$$

Now, put $x = 7$ in eq. (1), we get

$$3 \times 7 - 6 = y$$

or $$y = 21 - 6$$

or $$y = 15$$

$\therefore$ Required fraction $= \frac{x}{y} = \frac{7}{15}$

COMMON ERR(!)R

Some students confused between numerator and denominator of a fraction. So, adequate practice is required.

8. Let father's age be x and sum of ages of his children be y years.

TiP

Adequate practice should be needed on solving such type of word problem.

After 5 years, father's age will be $(x + 5)$ years and sum of ages of his children will be $(y + 10)$ years.

According to the given conditions,

Father's age = 3 × Sum of ages of his children

i.e. $x = 3y$ ... (1)

and after 5 years,

Father's age = 2 × Sum of ages of his children

i.e. $x + 5 = 2 \times (y + 10)$

$\Rightarrow x + 5 = 2y + 20$

$\Rightarrow x - 2y = 15$ ...(2)

From eq. (1), put the value of x in eq. (2), we get

$3y - 2y = 15 \quad \Rightarrow \quad y = 15$ years

Now put $y = 15$ in eq. (1), we get

$x = 3 \times 15 = 45$ years

Hence, present age of father is 45 years.

COMMON ERR(!)R

Some students frame this word problem into wrong equation, lead to get incorrect answer.

9. Let the speed of stream be x km/h.

$\therefore$ Speed of the boat rowing upstream

$= (5 - x)$ km/h

Speed of boat rowing downstream

$= (5 + x)$ km/h

TiP

Downstream means when we row with the flow of water so our speed increases, that's why we add both the speeds. Upstream means when we row against the flow of water so our speed decreases, that's why we subtract speed of current from speed of stream.

According to the question,

$$3 \times \frac{40}{5+x} = \frac{40}{5-x} \quad \left(\because \text{time} = \frac{\text{distance}}{\text{speed}}\right)$$

$\Rightarrow \frac{3}{5+x} = \frac{1}{5-x}$

$\Rightarrow 15 - 3x = 5 + x$

$\Rightarrow 4x = 10$

$\Rightarrow x = \frac{10}{4} = 2.5$

Hence, speed of the stream is 2.5 km/h.

Long Answer Type Questions

1.

TiP

An easy way of getting a solution of an equation is to take $x = 0$ and get the corresponding value of y. Similarly, we can put $y = 0$ and obtain the corresponding value of x.

For $6x - y + 4 = 0$, when $x = 0$, ...(1)

$6 \times 0 - y = -4$

$\Rightarrow y = 4$

and when $y = 0$, $6x - 0 = -4$

$\Rightarrow x = \frac{-4}{6} = -\frac{2}{3}$

x	0	–2/3
y	4	0

For $2x - 5y = 8$, when $x = 0$, ...(2)

$2 \times 0 - 5y = 8$

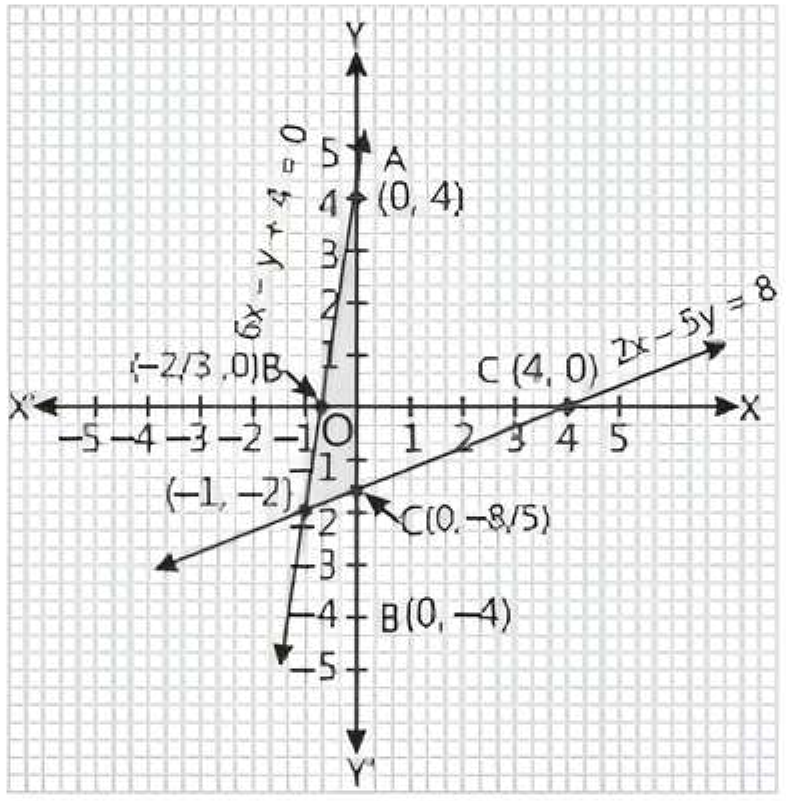
$\Rightarrow y = \frac{8}{-5} = -\frac{8}{5}$

and when $y = 0$, $2x - 5 \times 0 = 8$

$\Rightarrow x = \frac{8}{2} = 4$

x	0	4
y	–8/5	0

On a graph paper, draw X and Y-axes and plot the above points.



Both the lines intersect at (–1, –2).

Hence, (–1, –2) is the solution of given pair of linear equations.

2. Given equations are

$2x + 3y - 7 = 0$ ...(1)

and $(a - b)x + (a + b)y - (3a + b - 2) = 0$ ...(2)

Here, $\frac{a_1}{a_2} = \frac{2}{a-b}, \frac{b_1}{b_2} = \frac{3}{a+b}$

and $\frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)} = \frac{7}{(3a+b-2)}$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$

Take Ist and IIIrd terms, $\frac{2}{a-b}=\frac{7}{3a+b-2}$

$\Rightarrow 6a+2b-4=7a-7b \quad \Rightarrow \quad a-9b=-4 \quad$ ...(3)

Again take Ist and IInd terms, $\frac{2}{a-b}=\frac{3}{a+b}$

$\Rightarrow 2a+2b=3a-3b \quad \Rightarrow \quad a-5b=0 \quad$ ...(4)

On subtracting eq. (3) from eq. (4), we get

$$4b=4 \Rightarrow b=1$$

On substituting the value of 'b' in eq. (4), we get

$$a-5\times 1=0$$

$$\Rightarrow \quad a=5$$

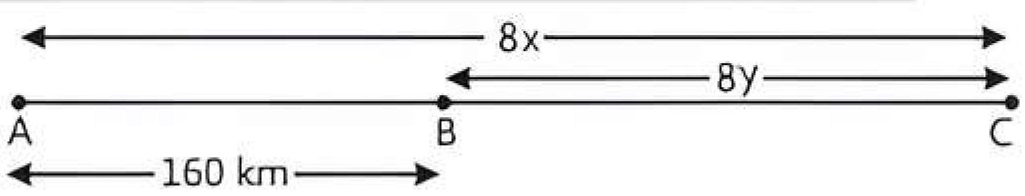
Hence, for $a=5$ and $b=1$, the given equations have infinitely many solutions.

3. Let the speed of car starting from place A be x km/h and that of another car starting from place B be y km/h.

TiP

Emphasis on solving such type of application based problem.

Case I: When two cars move in same direction.



Distance covered (AC) by first car in 8 h = $8x$

(∵ distance = time × speed)

Distance covered (BC) by second car in 8 h = $8y$

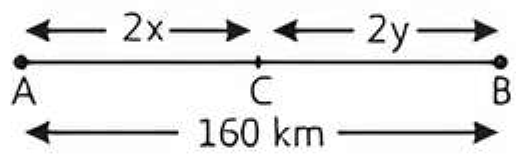
Now, $\quad AC-BC=160 \quad$ (given)

$\therefore \quad 8x-8y=160$

$\Rightarrow \quad 8(x-y)=160$

$\Rightarrow \quad x-y=\frac{160}{8}=20 \quad$ ...(1)

Case II: When two cars move in opposite directions.



Distance covered (AC) by first car in 2 h = $2x$

Distance covered (BC) by second car in 2 h = $2y$

Now, $\quad AC+CB=160$

$\therefore \quad 2x+2y=160$

$\Rightarrow \quad 2(x+y)=160$

$\Rightarrow \quad x+y=\frac{160}{2}=80 \quad$ ...(2)

On adding eqs. (1) and (2), we get

$$x-y=20$$
$$x+y=80$$
$$2x=100$$

$$\Rightarrow \quad x=\frac{100}{2}=50$$

On substituting the value of 'x' in eq. (1), we get

$$50-y=20$$

$\Rightarrow \quad -y=20-50=-30$

$\Rightarrow \quad y=30$

Hence, the speed of car that starts from place A is 50 km/h and that of another car starting from place B is 30 km/h.

COMMON ERR(!)R

Sometimes, students frame this word problem into wrong equation, lead to get incorrect answer.

Chapter Test

Multiple Choice Questions

Q 1. The pair of equations $3x-4y-11=0$ and $7x-5y-4=0$ has:

a. a unique solution

b. exactly two solutions

c. no solution

d. infinitely many solutions

Q 2. If the system of equations $3x+y=1$ and $(2k-1)x+(k-1)y=2k+1$ is inconsistent, then k is equal to:

a. 0   b. 2   c. −1   d. 1

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): If the pair of equations $x+2y+7=0$ and $3x+ky+21=0$ represents coincident lines, then value of k is 6.

Reason (R): The pair of linear equations are coincident lines, if they have no solution.

Q 4. Assertion (A): The graph of lines $4x+3y=12$ and $8x+6y=48$ is parallel.

Reason (R): The graph of linear equation $ax+b=0$, where $a\neq 0$ is parallel to Y-axis.

Fill in the Blanks

Q 5. Graphically, the pair of equations intersect at .... point.

Q 6. If the lines given by $3x+4ky=2$ and $2x+7y=1$ are parallel, then the value of k is ....

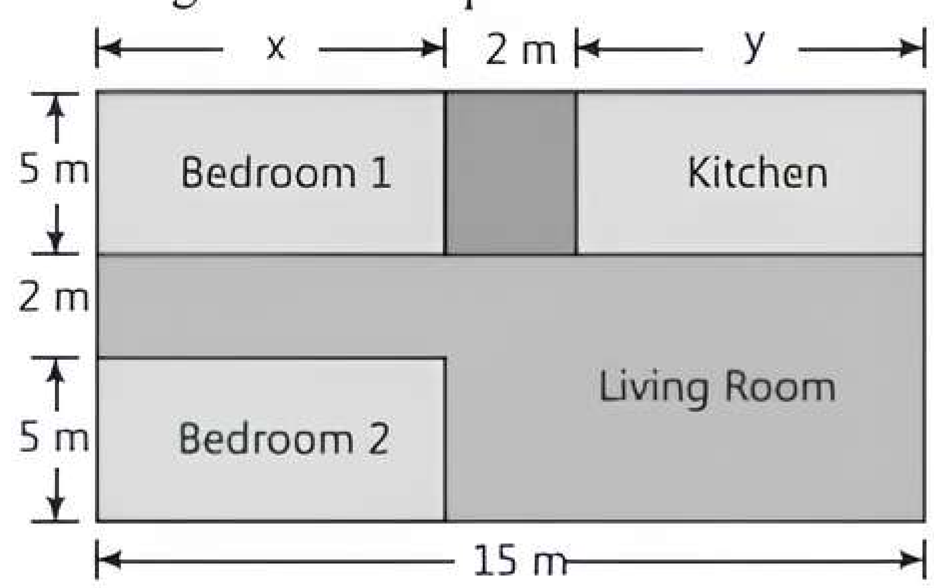
True/False

Q 7. The pair of equations $y = 0$ and $y = -7$ has no solution.

Q 8. The system of equations $ax + 3y = 1, -12y + ax = 2$ has unique solution for all real values of a.

Case Study Based Question

Q 9. Amit is planning to buy a house and the layout is given below. The design and the measurement has been made such that areas of two bedrooms and kitchen together is 95 sq. m.



Based on the given information, solve the following questions:

(i) Find the perimeter of the outer boundary of the layout.

(ii) Find the area of living room in the layout.

(iii) Find the area of each bedroom and kitchen in the layout.

OR

Find the area of living room in the layout.

Very Short Answer Type Questions

Q 10. Find the solution of the following equations:

$$2x + 3y = 17 \quad \text{and} \quad 3x - 2y = 6$$

Q 11. Show that system of equations $6x + 5y = 11$ and $9x + \frac{15}{2}y = 21$ has no solution.

Short Answer Type-I Questions

Q 12. The cost of 4 pens and 4 pencils boxes is ₹100. Three times the cost of a pen is ₹ 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

Q 13. Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages.

Short Answer Type-II Questions

Q 14. For which value(s) of λ does the pair of linear equations $\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have:

(i) no solution.

(ii) infinitely many solutions.

(iii) a unique solution.

Q 15. The sum of two numbers is 25 and their difference is 1. Find the numbers.

Long Answer Type Question

Q 16. A railway half ticket cost half the full fare but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the stations A to B costs ₹ 2530. Also, one full reserved first class ticket and one reserved first class half ticket from stations A to B costs ₹ 3810. Find the full first class fare from stations A to B and also the reservation charges for a ticket.